Average Difficulty

THIS ISSUE'S PUZZLE

At a recent dinner, a group of seven actuaries sitting around a table were, oddly enough, discussing actuary compensation issues. Although reticent to divulge their own individual annual compensations, they agreed that it would be useful if they knew the average salary of the group. As fate would have it, they had no paper or writing utensils of any sort to assist them, nor was anyone else available—yet eventually they derived a strategy that would enable themselves to know the group average, without anybody knowing the salaries of anybody else. How'd they do it?

Please submit your solution via email to PuzZzles@aol.com or by mail to:

PUZZLES, 17 Ravine Rd., Great Neck, NY 11023. Please submit answers as soon as possible to make the solvers list. Note: The solvers list will be limited to the first 40 solvers for this puzzle.

Please send any ideas, and new or old favorite puzzles that you think may be useful for future issues, to the same address or e-mail.

LAST ISSUE'S PUZZLE

Consider two decks of 52 regular playing cards:

Deck one is prearranged black-red-black-red, etc. throughout the deck that is placed face down.

It is shuffled in the following manner: 26 cards are drawn off, one at a time, and placed face down to create a new 26 card pile adjacent to the original (i.e. the original top card should now be the bottom card of the new pile). The two 26 card piles are then given one riffle shuffle.

Assume that in riffle shuffling cards, the number of alternating "grouped" cards taken from each pile follows the distribution: 3 cards probability 1/3, 2 cards probability 1/3, and 1 card probability 1/3—if at the end of a pile less than 3 cards are available then the distribution is 50/50 (2 or 1 card) where 2 cards are available, and obviously 100% where only 1 card is available.

Deck two is a deck that is thoroughly randomized. Cards are riffle shuffled one more time as above.

Question: Assume that the cards are now taken two at a time from the top of deck one. Suppose the precise mathematical expected number of pairs in which colors don't match is called X. Similarly, assume that cards are taken two at a time from the top of deck two. Call the precise mathematical expected number of pairs in which the colors don't match Y. What is X-Y?

Answer: 650/51 = 12 38/51 or approximately 12.7451

Solution:

DECK ONE—Tricky? This is actually the basis for a terrific card trick. Upon experimentation one can see that after any riffle shuffle the resulting pairs will be either red-black or black- red. [I leave the mathematical proof to the astute reader]. Thus the number of pairs with non-matching colors is necessarily equal to 26 and so X=26.

DECK TWO—each pair has an independent probability of not matching of 26/51, thus the expected number is Y= 26 x 26/51.

Note for mathematical magicians: there are various variations on the mathematical card trick discussed in the DECK ONE solution—one simple variation is to simply cut such a [secretly] prearranged red-black-red-black deck near the middle. After allowing spectator to cut deck, place bottom on top and have a spectator give a "good" single riffle shuffle. You then take deck back and cut again yourself, making it appear to be random—but actually splitting deck between any two like-colored adjacent cards, and then placing bottom cut on the top. After explaining that the probability of each pair not matching is approximately 50/50 [actually it is precisely 26/51] you show that 26 successive pairs do not match in color—an event which normally would have probability of approximately $(½)^{26}$.

This principal also applies to other attributes as well—for example, prearranging 52 cards as hearts, clubs, diamonds, and spades. After counting down 26, or any other number of cards as described, give a single riffle shuffle. Each 4-card group contains all four suits, and each pair does not match in color!

(I believe that variations of the DECK ONE trick were published by the famous recreational mathematics writer Martin Gardner and that he attributes authorship to another magician.)

July/August Solvers

F. Bernardi, W. Carroll, R. Dowsett, J. Edmondson, D. Glick, C. Jackson, D. Jacobs, J. Klemm, R. Olson, G. Strunk, G. Wang

September/October Solvers

B. Bartholomew, D. Bottelli, B. Erickson, T. Kelley, S. Peeples, D. Promislow, A. Spooner, D. Thaller, J. Young