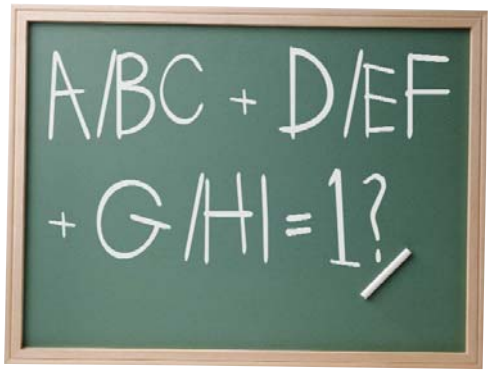
Mind your ABC's



THIS ISSUE'S PUZZLE

Actuaries, ever mindful of their p's and q's, may want to try minding their ABC's with this puzzle.

In the following expression each of the nine letters represents a different number—1 through 9. (The double digits are 2 digit numbers; i.e. the digits are NOT multiplied).

A/BC + D/EF + G/HI = 1

What number does each letter represent?

Thank you Chi Kwok for suggesting this issue's puzzle . (Apparently this puzzle was first composed by L. Mittenzwey for Mathematische Kurzweil.)

Please submit your answers via email to PuzZzles@aol.com or by mail to PUZZLES, 17 Ravine Rd., Great Neck, NY 11023. No solution is required. Please submit answers as soon as possible to make the solvers list. Please send any ideas, and new or old favorite puzzles that you think may be useful for future issues, to the same address or e-mail.

SOLUTIONS TO PREVIOUS PUZZLES

Confused and Confusing Passenger

Puzzle 1 (March/April). The original confused and confusing passenger problem stated that: The first of a line of eighty passengers gets on board a plane and disregards his/her reserved seat number, picking a seat randomly from among the 80 seats. Thereafter passengers boarding take their own seats—if already occupied they randomly select another seat. What is the probability that the last passenger sits in his own seat?

Answer is 1/2

A) 1st passenger has a 50/50 chance of taking his own seat or the last seat.

B) If the 1st confused passenger takes a different seat than his or last seat (which happens to occur at 78/80 probability) then a sequence of seat "bumpings" occurs—one person forcing another to look for a seat, and that person taking someone else's seat, et cetera, until the last of passengers 2 through 79, whose seat was taken by a previous person—either takes the last passenger's seat (and the last passenger consequently takes the first seat); or equally likely he/she takes the first passenger's seat (and last passenger takes his own seat)

C) Thus, since A and B are the only two possibilities, and in each case the last passenger has probability of 1/2 of sitting in his own seat, the answer must be 1/2.

Puzzle 2 (March/April). This has the same conditions as above, but assumes the first **two** passengers to get on board do not look at their tickets, and randomly select seats. Again, the remaining passengers getting on board select there own seats; however, if already occupied, they select another seat randomly. What is the probability that the last passenger sits in his own seat?

Answer is 1/3

Call the seat assigned to passenger n "seat n." Using a similar argument as used for puzzle 1, one can see that:

A) Of the situations where passenger 1 and passenger 2 pick their seats from among 1st, 2nd, and last seat—probability of last getting his own seat is 1/3.

B) Consider where both passenger 1 and 2 have picked from seats 3-79. Then 2 "bumper" sequences are created. Eventually the last of each resolves the sequence by picking either from seats 1, 2, or seat 80. Since the probability is random for the 2 (randomly selecting 2 out of 3), again the probability is 1/3 that last will get his own seat.

C) Similarly, one can analyze the last, somewhat subtler, situation. Here, either passenger 1 or passenger 2 initially selected seat 1 or 2 or seat 80, and the other (of passenger 1 and 2) initially picked from seats 3-79. Here again, as in B, the probability is 1/3 that last passenger will have his own seat. (Since the first picks randomly from among seats 1,2, and 80-and the last bumped from among the 3-79 picks from the remaining two.)

D) Thus since A, B, and C are the only possibilities-and each has probability of 1/3, the answer must be 1/3.

Puzzle 3 (May/June). Assume that a departing flight has the conditions of puzzle 1. It is given that the mathematical expected number of passengers not in their own seats, on this departing flight, is given by A.

On the return flight the first **four** passengers, in the same fashion, ignore their reserved seat numbers and select seats randomly. Again, the remaining passengers getting on board select there own seats; however, if already occupied, they select another seat randomly. Express the expected number of passengers that are not in their own seats on the return flight in terms of A.

Answer: 4A-13/3

Reflecting on the solution given for March/April puzzle one—the probability of last passenger sitting in his own seat on the departing flight is 1/2. For passenger 79—the probability of being in his own seat is 2/3 (think about the chain up to passenger 78). Similarly, we obtain probabilities of each sitting in his own seat as the following (beginning with 1st passenger) for the departing flight:

1/80, 79/80, 78/79, 77/78. . . 2/3, 1/2

The probability of not being in own seat is then 1 minus each of the terms, i.e.:

79/80, 1/80, 1/79,1/78. . . 1/4, 1/3,1/2.

A, being the expected value, of passengers not in their own seats, is simply equal to the sum of these terms. (A is approximately 4.913.)

Similarly, for the return flight the probability of getting correct seat is :

1/80,1/80,1/80,1/80,76/80,75/79. . . 3/7,2/6,1/5

The probability of not being in right seat is:

79/80,79/80,79/80,79/80, 4/80, 4/79, 4/78, . . 4/7,4/6,4/5.

The sum of these, is the expected number not in their own seats on return flight.

This sum is equal to

4A-4 (1/4+1/3+1/2) = 4A-13/3.

Obviously, the answers are independent of the 80-passenger assumption—e.g. the answers to Puzzles 1-3 do not change for 500 passenger flights.

January/February Solvers

Mark Evans

March/April Solvers

Michael Barth, Francis Bernardi, Eric Crane, Mark Evans, Jerry Francis, Jack Krull, Brian Miller, Brad Ober, Jeffrey Rykkus, Ronald Stokes, Kevin Trapp, Kuojen Yen

May/June Solvers

Gerry Anderson, Bob Bartholomew, Jim Chapman, Bill Cross, Mark Evans, Michael Failor, Carl Jared, Chi Kwok, Lee Michelson, David Promislow, Al Spooner, Kevin Trapp, Jenny Young